wave mode for a mixture of any number of components, without carrying out numerical calculations on a computer. The calculation sequence for the iterative method in the case of a three-component mixture with the parameters of the above example is shown in Fig. 2. The iteration is terminated when the difference between successive iterations is less than the given accuracy of the calculation.

NOTATION

c_m, concentration of m-th mixture component in gas–liquid flow; q_m, concentration of m-th component absorbed by the medium; ω_m, function describing the filling of the porous grain; g_m, function taking into account dependence of diffusion coefficient inside porous grain; γ_m^0, relative critical coefficient taking into account mass transfer on external boundary of porous grain; γ_m, relative critical coefficient taking into account mass transfer inside porous grain; α_{0m}, relative coefficient taking into account mass transfer due to longitudinal effective mixing; m_1, m_3, relative coefficients of heat transfer between gas flow and porous grains; m_2, relative coefficient of heat transfer with external surface of channel composed of porous grains; Q_m, relative thermal effect of sorption (desorption).

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STEADY-STATE TEMPERATURE FIELD OF A WALL WITH CYLINDRICAL COOLING CHANNELS

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The Bubnov–Galerkin method is combined with the structural method worked out by Rvachev to solve the problem of the steady-state temperature field of a wall with cylindrical cooling channels in a two-row arrangement.

We consider a flat wall ($-b \le y \le b$, $-\infty < x$, $z < \infty$), in which there are cylindrical cooling channels of radius r, arranged as in Fig. 1a. The wall material has a constant thermal conductivity λ. At the surfaces $y = \pm b$ the wall is heated by the surrounding gas which is at temperature T_1; the heat-transfer coefficient is α_1. This heat is transferred to the massive wall of the cooling liquid with temperature T_2; the heat-transfer coefficient of the surface of a channel containing liquid is α_2. We are to determine the steady-state temperature field of the wall. To do this, we combine the Bubnov–Galerkin method with the structural method worked out by Rvachev [1-3].

Making use of the symmetry of this unknown temperature field, we can reduce the problem to that of solving the Laplace equation in region Ω (Fig. 1a):

$$A\Theta = -\Delta\Theta = -\left(\frac{\partial^2\Theta}{\partial x_1^2} + \frac{\partial^2\Theta}{\partial x_2^2}\right) = 0,\ x = (x_1,\ x_2) \in \Omega \tag{1}$$

with the boundary conditions

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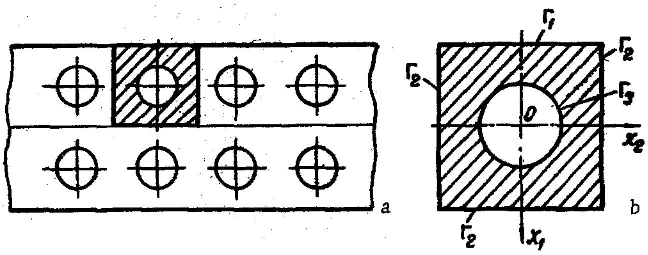

Fig. 1. a) Wall cross section; b) symmetric element of the wall.

$$\left[\frac{\partial\Theta}{\partial\nu_1}+\Theta\,\mathrm{Bi}_1\right]_{\Gamma_1}=h_1, \tag{2}$$

$$\left[\frac{\partial\Theta}{\partial\nu_2}-\mathrm{Bi}_2\,\Theta\right]_{\Gamma_2}=-h_2, \tag{3}$$

$$\left.\frac{\partial\Theta}{\partial\nu_3}\right|_{\Gamma_3}=0, \tag{4}$$

where

$$\Theta=\frac{T}{T_0};\ \Theta_1=\frac{T_1}{T_0};\ \Theta_2=\frac{T_2}{T_0};\ x_2=\frac{x}{\delta};\ x_1=\frac{y}{\delta};$$

$$\mathrm{Bi}_1=\frac{\alpha_1\delta}{\lambda};\ \mathrm{Bi}_2=\frac{\alpha_2\delta}{\lambda};\ h=\mathrm{Bi}_1\Theta_1;\ h_2=\mathrm{Bi}_2\Theta_2.$$

Here T_0 is some fixed temperature, δ is a scale dimension, and ν_k ($k = 1, 2, 3$) are the directions of the inward normals to the regions Γ_k of piecewise-smooth contour Γ.

To solve boundary-value problem (1)-(4) we construct the functions $\omega_k(x)$, which satisfy conditions (2):

$$\omega_k(x)\in C^2(\Omega),\ x\in\Omega, \tag{5}$$

$$\omega_k(x)>0,\ x\in\Omega, \tag{6}$$

$$\omega_k(x)=0,\ x\in\Gamma_k, \tag{7}$$

$$\frac{\partial\omega(x)}{\partial\nu_k}=1,\ x\in\Gamma_k. \tag{8}$$

Under the conditions of the present problem, the functions $\omega_k(x)$ are

$$\omega_1=x_1^2+x_2^2-1/4,\ \omega_2=1+x_2,$$

$$\omega_3=f_3+g_3-\sqrt{f_3^2+g_3^2},\ f_3=1-x_1,\ g_3=\frac{1-x_2^2}{2}.$$

We also introduce the differential operator D_k, defined by [2]

$$D_k=\frac{\partial\omega_k}{\partial x_1}\frac{\partial}{\partial x_1}+\frac{\partial\omega_k}{\partial x_2}\frac{\partial}{\partial x_2}. \tag{9}$$

According to (9), we have

$$\left.\frac{\partial\omega_k}{\partial x_1}\right|_{\Gamma_k}=\cos(\nu_k,\ x_1),\quad \left.\frac{\partial\omega_k}{\partial x_2}\right|_{\Gamma_k}=\cos(\nu_k,\ x_2), \tag{10}$$

so that we can write

$$D_kV|_{\Gamma_k}=\left.\frac{\partial V}{\partial\nu_k}\right|_{\Gamma_k},\ V\in C^2(\Omega). \tag{11}$$

We write the solution of boundary-value problem (1)-(4) as the expansion

$$\Theta=\Phi+\sigma_1\Phi_1+\sigma_2\Phi_2+\sigma_3\Phi_3, \tag{12}$$

where

$$\sigma_1=\frac{\omega_1(\omega_2\omega_3)^2}{\omega_1+(\omega_2\omega_3)^2};\ \sigma_2=\frac{\omega_2(\omega_1\omega_3)^2}{\omega_2+(\omega_1\omega_2)^2};\ \sigma_3=\frac{\omega_3(\omega_1\omega_2)^2}{\omega_3+(\omega_1\omega_2)^2}; \tag{13}$$

and $\Phi_s(x)$ ($s = 0, 1, 2, 3$) are certain arbitrary functions of the class $C^3(\Omega)$. Since we have

$$\sigma_l = \begin{cases} 0(\omega_k), & l = k; \\ 0(\omega_k^2), & l \neq k; \end{cases} \quad D_l\sigma_k = \begin{cases} 1 + 0(\omega_k), & l = k; \\ 0(\omega_k^2), & l \neq k, \end{cases} \tag{14}$$

we can write, using conditions (7) and (8) for $x \to x_0 \in \Gamma$,

$$\sigma_l \to 0, \quad D_l\sigma_k \to \begin{cases} 1 & l = k, \\ 0, & l = k. \end{cases} \tag{15}$$

It follows from conditions (7), (8), (11), and (15) that the function $\Theta(x)$, defined by expansion (12) satisfies boundary conditions (2)-(4) exactly for any choice of the functions $\Phi_s(x) \in C^2(\Omega)$.

With boundary condition (2) we now associate the functional-differential relation

$$D_1\Theta + \mathrm{Bi}_1\Theta = h_1 + \omega_1\psi_1, \quad \psi_1 \in C^2(\Omega). \tag{16}$$

By virtue of conditions (7) and (11), we can treat (16) as a continuation of boundary condition (2) into region Ω. Substituting expansion (12) into (16), we find

$$\begin{gathered} D_1\Phi_0 + \Phi_1 D_1\sigma_1 + \Phi_2 D_1\sigma_2 + \Phi_3 D_1\sigma_3 + \sigma_1 D_1\Phi_1 + \sigma_2 D_1\Phi_1 + \\ + \sigma_3 D_1\Phi_3 + \mathrm{Bi}_1(\Phi_0 + \sigma_1\Phi_1 + \sigma_2\Phi_2 + \sigma_3\Phi_3) = h_1 + \omega_1\psi_1. \end{gathered} \tag{17}$$

Using (14), we can rewrite (17) as

$$\begin{gathered} D_1\Phi_0 + \mathrm{Bi}_1\Phi_0 + \Phi_1 = h_1 + \omega_1\chi_1, \\ \chi_1 = \chi[\sigma_1, \sigma_2, \sigma_3; \Phi_1, \Phi_2, \Phi_3] \in C^2(\Omega). \end{gathered} \tag{18}$$

Solving (18) for the function $\Phi_1(x)$, we find

$$\Phi_1 = h_1 - D_1\Phi_0 - \mathrm{Bi}_1\Phi_0 + \omega_1\chi_1. \tag{19}$$

Analogously, using the functional-differential relations corresponding to boundary conditions (3) and (4), we find

$$\Phi_2 = h_2 - D_2\Phi_0 + \mathrm{Bi}_2\Phi_0 + \omega_2\chi_2, \tag{20}$$

$$\Phi_3 = -D_3\Phi_0 + \omega_3\chi_3. \tag{21}$$

After substituting (19)-(21) into (12), we find that Θ becomes

$$\begin{gathered} \Theta = \varphi_0 + \sigma_0\Phi_0 - \sum_{k=1}^{3} \sigma_k D_k\Phi_0 + \omega_k^2\chi_k, \\ \varphi_0 = h_1\sigma_1 - h_2\sigma_2, \quad \sigma_0 = 1 - \mathrm{Bi}_1\sigma_1 + \mathrm{Bi}_2\sigma_2. \end{gathered} \tag{22}$$

It is easy to show that the function $\Theta(x)$, defined by Eq. (22) satisfies boundary conditions (2)-(4) exactly if $\Phi_0(x) \in C^2(\Omega)$ and $\chi_k(x) \equiv 0$ [2, 3].

We can finally write the desired solution $\Theta(x)$ by means of a structural equation of the type

$$\Theta = \varphi_0(\sigma_1, \sigma_2; h_1, h_2) + u(\Phi_0), \tag{23}$$

where

$$u(\Phi_0) = \sigma_0\Phi_0 - \sum_{k=1}^{3} D_k\Phi_0.$$

The function $u(x)$ satisfies homogeneous boundary conditions exactly [regardless of the choice of the element $\Phi_0(x)$], and the function $\varphi_0(x)$ satisfies the corresponding inhomogeneous boundary conditions.

We write the element $\Phi_0(x)$ as the expansion

$$\Phi_0 = \sum_{i=1}^{n} C_i\lambda_i(x) \tag{24}$$

in terms of the functions $\lambda_i(x)$ of some system $\{\lambda_i\}_{i=1}^{\infty}$, which is complete with respect to the region Ω; we find an approximate solution for boundary-value problem (1)-(4):

$$\Theta_n = \varphi_0(x) + \sum_{i=1}^{n} C_i\varphi_i, \quad \varphi_i = \sigma_0\lambda_i - \sum_{k=1}^{3} \sigma_k D_k\lambda_i. \tag{25}$$

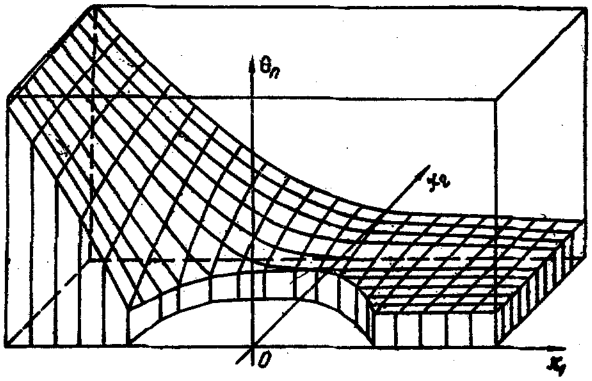

Fig. 2. Surface illustrating the temperature field of symmetric element of the wall.

The function $\Theta_n(x)$ satisfies boundary conditions (2)-(4) exactly for any choice of the constants C_i.

The unknown constants C_i are determined from the Bubnov–Galerkin system

$$\sum_{i=1}^{n} \alpha_{ij}\lambda_i = \beta_j \quad (j = 1, 2, \ldots, n), \; \alpha_{ij} = (\Delta\varphi_i - \Delta\varphi_0\varphi_j), \; \beta_j = (\Delta\varphi_0, \varphi_j).$$

Calculations have been carried out on an M-222 computer with $T_1 = 3500°K$, $T_2 = 393°K$; $T_0 = 400°K$, $b = 24 \cdot 10^{-3}$ m, $\alpha_1 = 5200$ W/(m^2 · deg), $\alpha_2 = 116{,}300$ W/(m^2 · deg), $\lambda = 3.49$ W/(m^2 · deg), $\delta = 12 \cdot 10^{-3}$ m, $\lambda_i = P_i(x)\Theta_i(x_2)$, $h = 0.05$, $n = 14$ [$P_i(x)$, $\Theta_i(x_2)$, ($|x_1|$, $|x_2| \leq 1$) are the Chebyshev polynomials]. The calculations yield the following values for the coefficients:

$$c_1 = 0.18482 \cdot 10; \; c_2 = 0.42392 \cdot 10^{-9}; \; c_3 = 0.25334 \cdot 10^{0};$$
$$c_4 = -0.14172 \cdot 10^{-9}; \; c_5 = -0.50125 \cdot 10^{-1}; \; c_6 = 0.22011 \cdot 10^{-10};$$
$$c_7 = 0.17009 \cdot 10^{-1}; \; c_8 = -0.12952 \cdot 10^{-11}; \; c_9 = -0.78927 \cdot 10^{-2};$$
$$c_{10} = 0.54548 \cdot 10^{-11}; \; c_{11} = 0.89502 \cdot 10^{-3}; \; c_{12} = -0.16802 \cdot 10^{-11};$$
$$c_{13} = 0.11798 \cdot 10^{-3}; \; c_{14} = 0.15103 \cdot 10^{-11}.$$

The calculation time required on the M-222 was 25 min.

The coefficients Ω were calculated for region C_i (Fig. 1a). The vanishing of the coefficients with even indices (the "computer zeros") is attributed to the symmetry of the desired temperature field with respect to the axis $0x_1$.

The approximate solution of boundary-value problem (1)-(4) can thus be written

$$\Theta_n = \varphi_0 + \sum_{i=1}^{n} C_i\varphi_i. \tag{26}$$

The approximate analytic solution of boundary-problem (1)-(4) was compared with the solution obtained by the electrical-analog method. The results agree satisfactorily, except near angular points which are instability zones of the calculation process.

Figure 2 shows a surface illustrating the steady-state temperature distribution in region Ω.

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